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Computational Experiments for the Problem of Hamiltonian Path with Fixed Number of Color Repetitions

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Abstract

In this paper we consider an approach to solve the problem of Hamiltonian path with fixed number of color repetitions for arc-colored digraphs. Our approach is based on usage of local search algorithms to solve a logical model for the problem.

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Let $G = (V; E)$ be a digraph where

$$V = \{v[1], v[2], ..., v[n]\}$$

is the set of nodes,

$$E = \{(v[i[1]], v[j[1]]), ...\}$$

is the set of arcs. We assume that each arc of G has a color. If the number of colors of arcs is restricted by an integer c, we speak about c-arc-colored digraphs. Let $C[(u,v)]$ be the color of (u,v). We can assume that

$$C[(u,v)] \in \{1,2,\ldots,c\}.$$

Let r be a fixed positive integer. Now, we consider the following problem.

THE PROBLEM OF HAMILTONIAN PATH WITH FIXED NUMBER OF COLOR REPETITIONS FOR c-ARC-COLORED DIGRAPHS:

INSTANCE: *A c-arc-colored digraph $G = (V; E)$.*

QUESTION: *Is G has a Hamiltonian path*

$$v[k[0]], v[k[1]], ..., v[k[n-1]]$$

such that

$$C[(v[k[i]], v[k[i+1]])] =$$
$$C[(v[k[i+1]], v[k[i+2]])] = ... = C[(v[k[i+r-1]], v[k[i+r]])],$$
$$|C[(v[k[j]], v[k[j+1]])] - C[(v[k[j+1]], v[k[j+2]])]| > 0,$$

for all

$$i = 0 \bmod r,$$
$$j+1 = 0 \bmod r,$$
$$i \in \{0,1,...,n-1\},$$
$$j \in \{0,1,...,n-1\}?$$

Note that different Hamiltonian path problems are extensively studied (see e.g. [1] – [3]). have natural applications in the design of experiments. In particular, the problem of Hamiltonian path with fixed number of color repetitions for c-arc-colored digraphs can be used for scenarios creation for self-learning of intelligent mobile robots.

Note that the problem of Hamiltonian path with fixed number of color repetitions for c-arc-colored digraphs is **NP**-complete. Encoding different hard problems as instances of SAT and solving them with efficient satisfiability algorithms has caused considerable interest (see e.g. [4] – [19]). In this paper, we consider an approach to solve the problem of Hamiltonian path with fixed number of color repetitions for c-arc-colored digraphs. Our approach is based on an explicit reduction from the problem to the satisfiability problem. We consider different local search algorithms to solve a logical model for the problem.

Let

$$\begin{aligned}
\varphi[1] &= \wedge_{0\le i\le n-1} \vee_{1\le j\le n} x[i,j],\\
\varphi[2] &= \wedge_{\substack{0\le i\le n-1,\\ 1\le j[1]<j[2]\le n}} (\neg x[i,j[1]] \vee \neg x[i,j[2]]),
\end{aligned}$$

$$\begin{aligned}
\varphi[3] &= \wedge_{\substack{0\le i[1]<i[2]\le n-1,\\ 1\le j\le n}} (\neg x[i[1],j] \vee \neg x[i[2],j]),\\
\varphi[4] &= \wedge_{\substack{0\le i\le n-2,\\ 1\le j[1]\le n,\\ 1\le j[2]\le n,\\ (j[1],j[2])\notin E}} (\neg x[i,j[1]] \vee \neg x[i+1,j[2]]),\\
\varphi[5] &= \wedge_{\substack{0\le i\le n-3,\\ 1\le j[1]\le n,\\ 1\le j[2]\le n,\\ 1\le j[3]\le n,\\ C[(j[1],j[2])]\neq C[(j[2],j[3])],\\ i+1\neq 0 \bmod r}} (\neg x[i,j[1]] \vee \neg x[i+1,j[2]] \vee \neg x[i+2,j[3]]),\\
\varphi[6] &= \wedge_{\substack{r-1\le i\le n-r-1,\\ 1\le j[1]\le n,\\ 1\le j[2]\le n,\\ 1\le j[3]\le n,\\ C[(j[1],j[2])]= C[(j[2],j[3])],\\ i+1= 0 \bmod r}} (\neg x[i,j[1]] \vee \neg x[i+1,j[2]] \vee \neg x[i+2,j[3]]),\\
\xi &= \wedge_{i=1}^{6}\varphi[i].
\end{aligned}$$

Theorem. *The function ξ is satisfiable if and only if there is a Hamilton path*

$$v[k[0]], v[k[1]], ..., v[k[n-1]]$$

in G such that

$$C[(v[k[i]], v[k[i+1]])] =$$

$$C[(v[k[i+1]], v[k[i+2]])] = ... = C[(v[k[i+r-1]], v[k[i+r]])],$$

$$|C[(v[k[j]], v[k[j+1]])] - C[(v[k[j+1]], v[k[j+2]])]| > 0,$$

for all

$$i = 0 \bmod r,$$

$$j+1 = 0 \bmod r,$$

$$i \in \{0, 1, ..., n-1\},$$

$$j \in \{0, 1, ..., n-1\}.$$

Proof. Let $\xi = 1$. In view of the definition of ξ, in this case, it is clear that $\varphi[i] = 1$, for all $1 \le i \le 6$.

Since $\varphi[1] = 1$, it is easy to see that for all $0 \le i \le n-1$, there is $1 \le j \le n$ such that $x[i,j] = 1$. In view of $\varphi[2] = 1$, it is clear that for any $0 \le i \le n-1$, there is only one value of j such that $x[i,j] = 1$. Let $x[i, a[i]] = 1$. Since $\varphi[3] = 1$, it is easy to see that $a[i] \neq a[j]$, for any $i \neq j$. Therefore, values of

$x[i,j]$ can be considered as a choice of nodes for a path $y[0], y[1], \ldots, y[n-1]$. In particular, we can assume that $v[j]$ is the ith node in the path if and only if $x[i,j] = 1$.

Clearly, $\varphi[4] = 1$ if and only if the sequence

$$y[0], y[1], \ldots, y[n-1]$$

of nodes is a path. Since $\varphi[5] = 1$, it is easy to see that

$$C[(y[i], y[i+1])] = C[(y[i+1], y[i+2])] = ... = C[(y[i+r-1], y[i+r])],$$

for all

$$i = 0 \bmod r,$$

$$i \in \{0, 1, ..., n-1\}.$$

In view of $\varphi[6] = 1$, we obtain that

$$|C[(y[j], y[j+1])] - C[(y[j+1], y[j+2])]| > 0,$$

$$j + 1 = 0 \bmod r,$$

$$j \in \{0, 1, ..., n-1\}.$$

Now, we assume that there is a Hamilton path

$$v[k[0]], v[k[1]], ..., v[k[n-1]]$$

in G such that

$$C[(v[k[i]], v[k[i+1]])] =$$

$$C[(v[k[i+1]], v[k[i+2]])] = ... = C[(v[k[i+r-1]], v[k[i+r]])],$$

$$|C[(v[k[j]], v[k[j+1]])] - C[(v[k[j+1]], v[k[j+2]])]| > 0,$$

for all

$$i = 0 \bmod r,$$

$$j + 1 = 0 \bmod r,$$

$$i \in \{0, 1, ..., n-1\},$$

$$j \in \{0, 1, ..., n-1\}.$$

Let $x[i,j] = 1$ if and only if $j = k[i]$, for all $0 \leq i \leq n-1$. It is easy to check that $\xi = 1$. $\square$

Using standard transformations (see e.g. [20]) we can easily obtain an explicit transformation ξ into ζ such that $\xi \Leftrightarrow \zeta$ and ζ is a 3-CNF. It is clear that ζ gives us an explicit reduction from the problem of Hamiltonian path with fixed number of color repetitions for c-arc-colored digraphs to 3SAT.

We have created a generator of natural instances for the problem of Hamiltonian path with fixed number of color repetitions for c-arc-colored digraphs. There is a well known site on which posted solvers for SAT [21]. We have used algorithms from [21]: fgrasp and posit. For solution of the problem, we have used a heterogeneous cluster. Each test was run on a cluster of at least 100 nodes. Note that due to restrictions on computation time (20 hours) we used savepoints. Selected experimental results are given in Table .

time	average	max	best
fgrasp	19.74 hr	56.04 hr	27.12 min
posit	12.63 hr	37.25 hr	31.08 min

Table 1: Experimental results for 3SAT

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